Université Lumière Lyon 2

Faculté des Sciences Economiques

Sources of Errors and Biases in Traffic Forecasts for Toll Road Concessions

Thèse pour le Doctorat ès Sciences Economiques

Mention Economie des Transports

Antonio NUNEZ

dirigée par M. le Professeur Alain BONNAFOUS

Présentée et soutenue publiquement le
5 décembre 2007.

Membres du Jury:

M. Alain BONNAFOUS	Pr. à l'IEP de Lyon	*Directeur*
M. Yves CROZET	Pr. à l'Université Lyon 2	
M. Jean DELONS	Chargé de Mission à Cofiroute	
M. Fabien LEURENT	Pr. à l'ENPC	
M. Werner ROTHENGATTER	Pr. à l'Université de Karlsruhe	*Rapporteur*
M. Stéphane SAUSSIER	Pr. à l'Université de Paris 11	*Rapporteur*

Contents

List of Figures

List of Tables

Chapter 4

Decreasing Long-Term Traffic Growth

-Estimating the Functional Form of Road Traffic Maturity- [1] [2]

> "A straight line may be the shortest distance between two points, but it is by no mean the most interesting." (Doctor Who)

[1] I gratefully acknowledge comments and suggestions from Jean Delons, Elisia Engelmann, Homero Neves, an anonymous referee of the Journal of Spatial and Network Economics, and two anonymous referees and participants of the 24th Pan-American conference on Traffic and Transportation Engineering, Spain, September 20-23 2006.

[2] A version of this chapter was accepted for publications in the Journal of Spatial and Network Economics.

Abstract

It has been observed that motorways with high traffic levels experience lower traffic growth than those with lower traffic (ceteris paribus). This phenomenon is known as traffic maturity; however, it is not captured through traditional time-series long-term forecasts, due to constant elasticity to GDP these models assume, leading overestimation in traffic forecasting for these motorways. In this chapter we argue that traffic maturity results from decreasing marginal utility of transport. The elasticity of individual mobility with respect to the revenue decreases after a certain level of mobility is reached. In order to find evidences of decreasing elasticity we analyse a cross-section time-series sample including 40 French motorways' sections. This analysis shows that decreasing elasticity can be observed in the long term. We then propose a decreasing function for the traffic elasticity with respect to the economic growth, which depends on the traffic level on the road. This model seems to well explain the observed traffic evolution and gives a rigorous econometric approach to time-series traffic forecasts, producing more accurate forecasts.

4.1 Introduction

The link, or coupling, between traffic and economic growth is a strong concept in transport and regional planning. In aggregated models of transport demand forecast, individual mobility and revenue are represented by traffic and gross domestic product (GDP). Mobility generates traffic and we suppose that growth in GDP leads to growth in purchase power. In economics, this link is represented by an elasticity of traffic with respect to the GDP, usually greater than one. We can observe that older high traffic motorways experience lower traffic growth than newer, low traffic, ones (*ceteris paribus*). This phenomenon is known as traffic maturity in analogy with market maturity, a well known stage of products lifecycle. This phenomenon is not captured through traditional time-series long-term forecasts, due to constant elasticity to GDP these models assume. However, the observation of long traffic growth series put in evidence a growth deceleration in the long term.

In this sense we argue that the application of traditional traffic forecast models using time series with constant elasticity of traffic with respect to the GDP produces high growth hypothesis, leading to traffic overestimation. This study aims at putting in evidence a decreasing relationship between the traffic lever and the elasticity of the traffic with respect to economic growth and proposes a new econometric formulation for the time-series traffic forecast which considers the elasticity of traffic with respect to the GDP as a function of traffic level. Results show that this new model produces more reliable and precise forecasts.

The chapter is organized as follows: section 2 presents the stages of traffic growth and the traditional econometric approach. Section 3 proposes that traffic maturity is a direct consequence of the decreasing marginal utility of transport. In section 4 we present the Partial Adjustment Model and the Error Correction Model. Section 5 puts in evidence the decreasing of elasticity over the traffic lever using data from 40 cross-sections time series sample. Section 6 proposes the new model and shows the impact in long term forecasts. Section 7 briefly concludes the chapter.

4.2 Traffic Growth

In transport demand forecast, whether for road, rail or air link, three growth stages are identified: the ramp-up, the traffic growth and the maturity. Ramp-up describes the delay traffic needs to reach its market share. The ramp-up period reflects the users' lack of familiarity with the new infrastructure and its benefits. It can also be due to reluctance to pay tolls or to information lags. The ramp-up period is characterized by a high traffic growth, from a level that is lower than expected as the equilibrium.

Another important phenomenon affecting the ramp-up is the induced traffic. Induced traffic is the increment of new vehicle traffic resulting from a road capacity improvement. It represents the latent demand, excluding shifts from other modes or routes, changing in departure time and longer distances (which account for induced travels) and exogenous factors (as growth in population and economy). New trips to existing locations, trips that would not have occurred otherwise, are the purest form of induced traffic (Goodwin, 1996; Mokhtarian et al., 2002).

As the short term impacts get over, the traffic evolution results from the growth in demand, which comes from the economic and population growths and the impact of monetary costs (toll, fuel and operating costs) on the route chosen and on alternative routes and modes. After a certain level is reached, traffic grows slower, giving evidence that the need for transport was satisfied. Disregarded in transport, market maturity is nevertheless a main issue in new products market analysis, for which the life cycle is shorter and concurrence stronger than in transport sector. In the transport sector, this phenomenon has been recognized and studied at first in the air transport for tourism (Department for Transport, 1997; Graham, 2000); the possibilities to go on holidays been constrained, we should expect traffic will not grow unlimitedly.

The volume of traffic on a motorway can be assumed to depend on the level of economic activity, on the monetary and time costs of the motorway and on those of the alternative route and modes, as well as on the transport system characteristics. Monetary cost is defined as the sum of three components: toll, fuel price and other vehicle operating costs. Besides, given that demand for transport is a derived demand, other variables that have an effect on traffic should also be included in the equation. In this case, traffic volume in a specific motorway section is assumed to depend on the capacity of traffic emission and attraction of origins and destinations. The model can therefore be expressed as follows (Matas and Raymond, 2003):

$$T_{it} = \alpha_{0i} + \alpha_{1i}GDP_t + \alpha_{2i}PF_t + \alpha_{3i}Toll_{it}^M + \alpha_{4i}VC_{it}^M + \alpha_{5i}TC_{it}^M + \alpha_{6i}VC_{it}^R + \alpha_{7i}TC_{it}^R + \alpha_{8i}E_i + \alpha_{9i}A_i + \varepsilon_{it} \quad (4.1)$$

where

T_{it} is the traffic volume at the motorway section i and period t,

GDP_t is the level of economic activity in period t,

PF_t is the fuel price in period t,

$Toll_{it}$ is the motorway toll in section i and period t,

VC_{it}^j are other vehicle operating costs, $j = M, R$ refers to motorway and alternative modes, respectively,

TC_{it}^j are the time costs in section i and period t,

E_i is the emission factor in section i,

A_i is the attraction factor in section i.

However, in the context where this estimation takes place it can be assumed that other vehicle operating costs and time costs remain constant over time. Thus, it is assumed that VC_{it}= VC_i and TC_{it}= TC_i. Therefore, after substitution, we get:

$$T_{it} = [\alpha_{0i} + \alpha_{4i}VC_{it}^M + \alpha_{5i}TC_{it}^M + \alpha_{6i}VC_{it}^R + \alpha_{7i}TC_{it}^R + \alpha_{8i}E_i + \alpha_{9i}A_i] + \alpha_{1i}GDP_t + \alpha_{2i}PF_t + \alpha_{3i}Toll_{it}^M + \varepsilon_{it} \quad (4.2)$$

Thus, the demand equation can be re-written as:

$$T_{it} = \beta_{0i} + \alpha_{1i} GDP_t + \alpha_{2i} PF_t + \alpha_{3i} Toll_{it}^{M} + \varepsilon_{it}$$

where β_{0i} captures the terms in brackets in equation (4.2). This equation is usually applied on the log-log form. This transformation reduces heteroscedasticity and gives a convenient interpretation of results, which can be read directly as elasticities. The equation becomes:

$$lnT_{it} = \beta_{0i} + \alpha_{1i} lnGDP_t + \alpha_{2i} lnPF_t + \alpha_{3i} lnToll_{it}^{M} + \varepsilon_{it} \tag{4.3}$$

This model, henceforth called LTM, for long-term model, represents a long-term equilibrium between the variables. The elasticity of traffic with respect to the GDP in section i is α_1 because:

$$\varepsilon_{T/GDP} = \frac{GDP}{T}\frac{\delta T}{\delta GDP} = \frac{\delta lnT}{\delta lnGDP} = \alpha_1 \tag{4.4}$$

This constant elasticity specification is generally used in empirical studies but it is however questionable since we could expect the elasticity to be decreasing; this argument is developed in the next section.

4.3 Why does Traffic Grow Decreasingly?

The consumer theory, from its classic axioms, transforms preferences in utility. The law of decreasing marginal utility states that marginal utility decreases as the quantity consumed increases. In essence, each additional good consumed is less satisfying than the previous one. This law holds for most goods, and do so for transport. This principle supports the idea of decreasing transport growth since the utility of an additional travel depends on individual's mobility. Furthermore, time and money constraints limit transport possibilities.

New traffic comes from new users on the route or mode and from existent users making more or longer trips. The traffic increment due to new users results from population growth as well as changes in land use and in locations of economic activities. Furthermore, reductions in transport costs as well as increases in user's wealth allow people to travel more and more often. This is particularly evident in the case of the air transport sector, where price reductions due to competition in the last years had not only diverted users from other modes but also allowed less rich people to afford air travels.

For existing users, the reduction on generalized costs, increasing in wealth and reduction and flexibility in working time allow users to travel more often. The possibility of supplementary trips is however constrained by time (daily time and holidays) and money availability. Budget and time depend not only on transport itself but on time and money spent in all others activities. These constraints unequally affect different people and different population classes. A retired person is supposed to be more constrained by money than by time, inversely to a rich businessman.

In addition to budget and time constraints, there is the will to travel. We can reasonably suppose that the higher is the individual's mobility level, the lesser will be his inclination or necessity to make one more trip. Despite regular fluctuations in transport demand, i.e. seasonal peaks, it has been suggested (for example, by Thomson (1974)) that over time, there has been a remarkable stability in the demand for travel, with households, for example, on average making roughly the same number of trips during a day albeit for different purposes or by different modes. There may be more leisure travel, but there are fewer work trips and greater is now made of air transport and the motorcar at the expense of walking and cycle. It is suggested that this situation reflects the obvious fact that there is a limit to the available time people have for travel, especially if they are to enjoy the fruits of the activities at the final destinations (Button, 1993).

This phenomenon is formulated as the decreasing marginal utility of travel, which means that $U(t) > 0$, $U'(t) > 0$ and $U''(t) < 0$, where $U(t)$ is the utility of transport. The utility function and constraints compose the individual's utility maximization program, where individual make trade-offs between possible allocations of resources. Utility functions define choices which generate demand functions, from which elasticities can be derived. Elasticities give adimensional measures of sensibility of a variable with respect to another. Elasticities are then concise measures of preferences and reflect the sensibility to changes in a limited resources environment (figure 4.1).

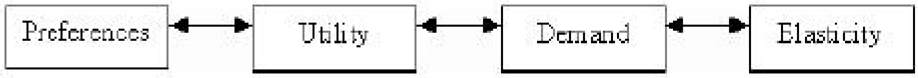

Figure 4.1: From preferences to elasticity.

The ordinary or Marshallian demand function is derived from consumers who are postulated to maximize utility subject to a budget constraint. As a good's price changes, the consumer's real income (which can be used to consume all goods in the choice set) changes. In addition the goods price relative to other goods changes. The changes in consumption brought about by these effects following a price change are called *income* and *substitution* effects respectively. Thus, elasticity values derived from the ordinary demand function include both income and substitution effects (Gillen et al., 2004).

In this sense, the elasticity of individual mobility with respect to the revenue decreases after a certain level of mobility is reached. In aggregated terms, the superposition of individual behaviours results in an increment in traffic which is decreasing in the part of traffic generated by existing users and therefore for economic and population constant growth, globally decreasing.

Congestion also constrains traffic growth. It has a double effect, first it physically limits traffic growth and second it reduces the generation of traffic by increasing the generalised cost. Nevertheless, traffic maturity must be isolated of congestion. Traffic maturity is a pure demand effect while congestion comes from the interaction of a level of demand higher then infrastructure capacity. We argue that maturity do not depends on supply (while traffic does). This argument is valid if we consider that congestion is limited to special periods (holiday departure) or a particular OD pair, affecting at the individual level, while our analysis focuses in a more aggregated level.

4.4 Econometric Issues

4.4.1 Partial Adjustment

The model (4.3) implies a long-run relationship between the variables; in any given period, actual demand could only be expected to be in equilibrium with (and so to be completely explained by) the income and costs associated in each period. However, the persistence of habit, uncertainty and incomplete information are some reasons why complete adjustment could not be achieved in a single period. In this case, the desired demand in year t, T^*_{it} is not equivalent to the actual demand in t, T_{it}. Although behavioural adjustment is toward the equilibrium, only a proportion, θ, of the gap between the desired (equilibrium) demand and actual demand is closed each year. This can be written as:

$$T_{it} - T_{it-1} = \theta(T^*_{it} - T_{it-1}) \tag{4.5}$$

where θ $(0 \leq \theta \leq 1)$ is the adjustment coefficient, which indicates the rate of adjustment to long term equilibrium and reflects the inertia of economic behaviour. Rearranging (4.5) and substituting in (4.3) we obtain the following Partial Adjustment Model:

$$lnT_{it} = \theta\beta_{0i} + \theta\alpha_{1i}lnGDP_t + \theta\alpha_{2i}lnPF_t + \theta\alpha_{3i}lnToll^M_{it} + (1-\theta)lnT_{it-1} + \varepsilon_{it} \tag{4.6}$$

or equivalently:

$$lnT_{it} = \beta_{0i} + \alpha_{1i}lnGDP_t + \alpha_{2i}lnPF_t + \alpha_{3i}lnToll^M_{it} + \phi lnT_{it-1} + \varepsilon_{it} \tag{4.7}$$

where the short-run elasticities are given by the coefficients α's and the long-run elasticities are the ratio of the short-run value by 1-ϕ.

4.4.2 Integrated variables, Cointegration and Error-Correction

Most time-series techniques need data to be stationary, but this requirement is often not fulfilled by economic series, which tend to increase over time. Those problems were somehow ignored in applied work until important papers by Granger and Newbold (1974) and Nelson and Plosser (1982) alerted many to the econometric implications of non-stationarity and the dangers of running *nonsense* or *spurious* regressions.

A non-stationary series can be made stationary by detrending series. A convenient way of detrending is by using first differences rather than levels of the variables. A non-stationary series which can be made stationary by differencing d times is said to be integrated of order d, denoted $x_t\tilde{I}(d)$, a stationary series is a $I(0)$ series (Engle and Granger, 1987).

While removing trending by differencing can actually be a statistical satisfactory solution, it represents a lost of economic information about the long term relationship. However, for some time it remained to be well understood how both variables in differences and levels could coexist in regression models. (Granger, 1981), resting upon the previous ideas, solved the puzzle by pointing out that a vector of variables, all of which achieve stationarity after differencing, could have linear combinations which are stationary in levels. Later, (Engle and Granger, 1987), were the first to formalize the idea of integrated variables sharing an equilibrium relation which turned out to be either stationary or have a lower degree of integration than the original series. They denoted this property by *cointegration*, signifying co-movements among trending variables which could be exploited to test for the existence of equilibrium relationships within a fully dynamic specification framework. In this sense, the basic concept of cointegration applies in a variety of economic models. A humorous illustration of this concept is given by Murray (1994) and extended by Harrison and Smith (1995).

Before proceeding with the cointegration analysis, it is necessary to verify whether the variables under consideration are stationary, and if not, check their orders of integration. This can be accomplished using the unit-root test. The most widely used unit-root test is the Augmented Dicky-Fuller (ADF) test, which involves running (with constant, trend and p lags):

$$\Delta y_t = \mu + \beta_t + \gamma y_t + \sum_{j=1}^{p} \phi_j \delta y_{t-1} + \varepsilon_t \tag{4.8}$$

This test was applied for each section as well as for the independent variables. The null hypothesis of unit root was always non-rejected (tables 4.1 and 4.2).

Various methods have been suggested to test for cointegration. One method is to estimate the long-run relationship (as in (4.3)) by OLS and testing whether the residual is stationary. This can be done using the Durbin-Watson statistic, DF or ADF tests. The hypothesis of unit roots of residuals could always be rejected .

Table 4.1: ADF test - exogenous variables

	Variables (in logarithms)	
	adf	p-value
GDP	-3.3579	0.08363
Fuel	-2.8059	0.2654
Toll 1	-2.3442	0.4412
Toll 2	-4.1275	0.0188
Toll 3	-2.3482	0.4397
Toll 4	-1.8115	0.6442
Toll 5	-2.0474	0.5543
Toll 6	-3.3201	0.0888
Toll 7	-1.4157	0.7950

It should be stressed that unit-root tests in general do not produce unambiguous results. They are large sample tests and their behaviour in small samples is questionable. Moreover, the results of different tests are contradictory many times. Given these problems, any results regarding the stationarity or non-stationarity of a particular series must be treated with caution (Dargay et al., 2002). Furthermore, the link between the economic and traffic growth in not to be proved anymore.

According to the Granger Representation Theorem, cointegrated series can be represented by an Error Correction Model. The dependent variable in an Error-Correction Model (ECM) is specified in terms of differences, rather than levels. ECM are well suited in cointegrated relationships since they incorporate the long-run relationships as well as the dynamics implied by the deviations from this equilibrium path and the adjustment process to recover it. The ECM can be written as (Dargay et al., 2002):

$$\Delta T_t = \alpha_0 + (\varphi - 1)T_{t-1} + \beta_0 \delta X_t + (\beta_0 + \beta_t)X_{t-1} + \varepsilon_t \tag{4.9}$$

where X is the vector of explanatory variables. More general forms could include higher order lagged differenced terms of the independent variables and lagged differences of the dependent variables. The model (10) can alternatively be written as:

$$\Delta T_t = \alpha_0 + \beta_0 \delta X_t + (\varphi - 1)\left[T_{t-1} + \frac{(\beta_0 + \beta_t)}{X_{t-1}} X_{t-1}]\right] + \varepsilon_t \tag{4.10}$$

The parameter β_0 represents the short-term effect and $(1 - \varphi)$ is the feedback effect, which is similar to the adjustment coefficient, θ, in the Partial Adjustment Model. The long-run response is given by $(\beta_0 + \beta_1)/(1 - \varphi)$. The term in the square brackets in equation (A5) is called an "error-correction mechanism" since it reflects the deviation from the long run, with $1 - \varphi$ of this deviation being closed each period. The Error Correction Model allows estimation of both short- and long-run parameters simultaneously. If the error-correction term $\varphi - 1$ is significantly different from zero and negative (since $0 < \varphi < 1$) the variables are cointegrated and the estimated parameters of the lagged level variables define the long-run relationship. The estimated model then takes the following form:

$$\begin{aligned} \Delta lnT_{it} = {} & \beta_{0i} + \beta_{1i}\Delta lnGDP_t + \beta_{2i}\Delta lnPF_t + \beta 3i\Delta lnToll_{it}^M + \\ & \alpha_{1i}lnT_{it-1} + \alpha_{2i}lnGDP_{t-1} + \alpha_{3i}lnPF_{t-1} + \alpha_{4i}lnToll_{it-1}^M + \varepsilon_{it} \end{aligned} \tag{4.11}$$

4.5 Data and Estimation

The data used in this analysis comes from the ASFA (Federation of French motorways concessionaires). Our sample includes 40 French motorway's sections with traffic series longer than 15 years, in different French regions and including all the main concessionaires (ASF, APRR, COFIROUTE, SANEF and SAPN). The GDP series comes from the INSEE (National Institute for Statistics and Economic Studies). The series of toll prices for all concessionaires were provided by the the Department of Traffic and Economic Studies of COFIROUTE.

For each section and each model (LTM, PAM and ECM), we begin with a general specification which includes all explanatory variables, and proceed to exclude those which are either implausible because of magnitude or sign or insignificant in a statistical sense. All estimates and statistical tests presented in this chapter were computed using *SAS* v9.

4.6 Evidences of Decreasing Growth

A concavity can be observed for the last periods in many long term traffic series. Figure (4.2) and Figure (4.3) show this decreasing of growth in two French motorways. The issue here is to understand whether this deceleration of the growth indicates that the maturity had been reached or it results from an economic deceleration, an increasing in fuel costs or other factors.

In order to find evidences that this decreasing growth results from a decreasing elasticity we proceed to a three steps analysis. First, we estimate the long-run elasticity of traffic with respect to the GDP using the three models presented earlier. Second, we test for the statistical stability of parameters on these sections using the CUSUM2 tests. Finally, we segment the sample in order to observe the evolution of elasticities.

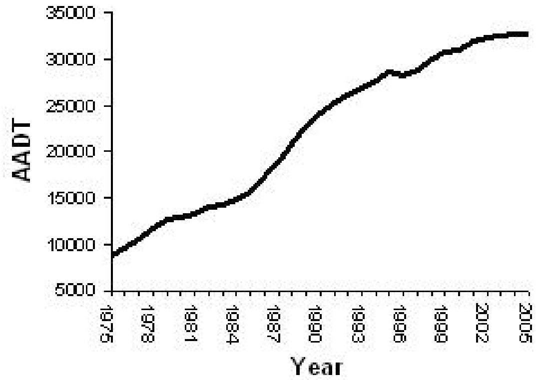

Figure 4.2: Traffic on the A10 motorway.

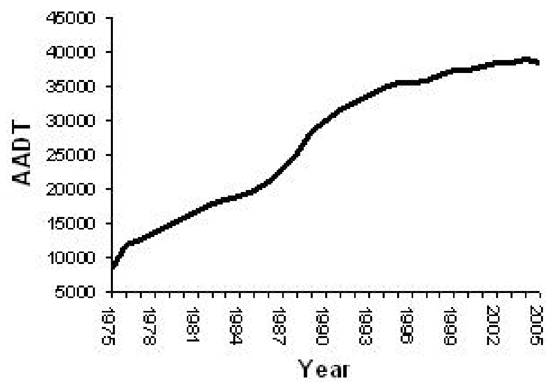

Figure 4.3: Traffic on the A11 motorway.

4.6.1 Cross-section Time Series Analysis

We applied the LTM, PAM and ECM for the 40 sections in order to determine the (constant) elasticity of traffic with respect to the GDP (results are presented in appendix 1). Plotting the long-run elasticity of the traffic with respect to the GDP over the traffic level in the first period (max(1980, opening date)) we can observe a clear decreasing relationship, i.e. sections with a high traffic at opening present a lower elasticity.

This result is however much less evident for the short-run elasticities. Some decreasing relationship can be found using the ECM but not with the PAM, moreover, many short-run elasticities are not statistically significant. This

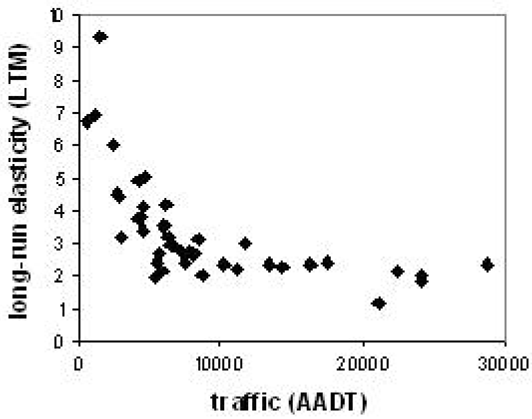

Figure 4.4: LTM long-run elasticities.

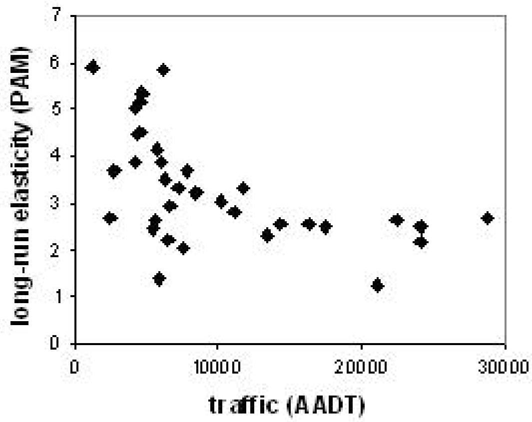

Figure 4.5: PAM long-run elasticities.

result can be viewed in figure (4).

An interesting issue here is to see whether the three models produce comparable elasticities. Comparing the statistical significant (at 90% level) long-run elasticities estimated by the LTM, PAM and ECM (appendix 1) we can see that (i) results are quite close in the three models for most sections and (ii) it seems that, in average, the PAM tends to produce slightly higher elasticities than the other models. Despite its incapacity of estimating short-run elasticities the LTM has the strong advantage of allowing for more robust estimates. It is the only model which produces statistical significant elasticities for every

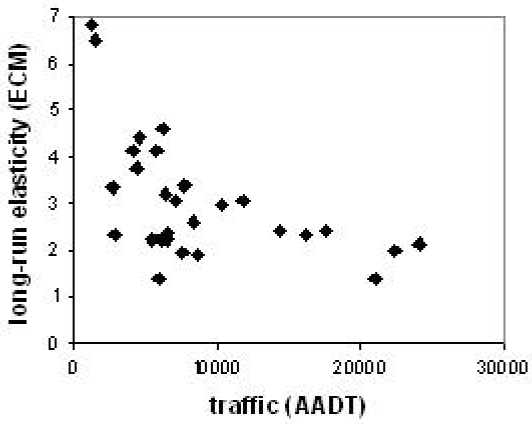

Figure 4.6: ECM long-run elasticities.

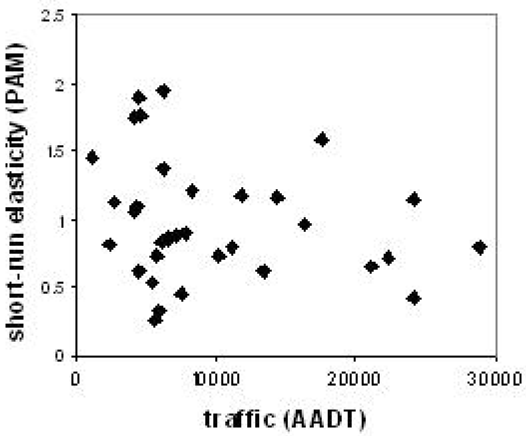

Figure 4.7: PAM short-run elasticities.

section.

4.6.2 Testing for Parameter Stability

Proposed by Brown et al. (1975) the CUSUM^2 (or CUSUM of squares) test for the constancy over time of the coefficients of a linear regression model. This tests is based on recursive residuals. The technique is appropriate for time series data and might be used if one is uncertain about when a structural change might have taken place (contrary to the Chow test). The null hypothesis is

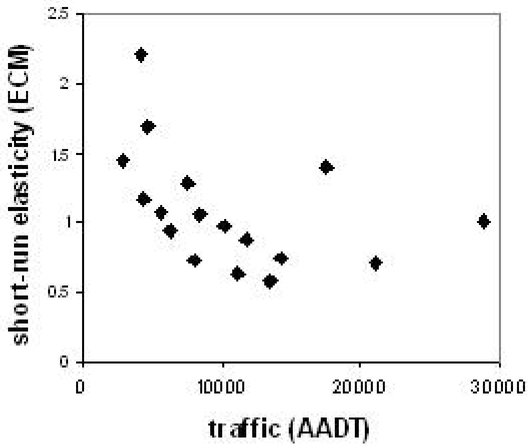

Figure 4.8: ECM short-run elasticities.

that the coefficient vector β is the same in every period; the alternative is simple that it (or the disturbance variance) is not. The test is quite general in that it does not require a prior specification of when the structural change takes place and is preferred to the CUSUM due to its higher power.

Suppose that the sample contains a total of T observations. The tth recursive residual is the ex-post prediction error for y_t when the regression is estimated using only the first $t-1$ observations. Since it is computed for the next observation beyond the sample period, it is also labeled a one step ahead prediction error;

$$e_t = y_t - x_t'\hat{\beta}_{t-1}$$

where x_t is the vector of regressors associated with the observation y_t and $\hat{\beta}_{t-1}$ is the least square coefficients computed using the first $t-1$ observations. The forecast variance of this residual is:

$$\sigma_{ft}^2 = \sigma^2[1 + x_t'(X_{t-1}'X_{t-1})^{-1}x_t]$$

Let the rth scaled residual be

$$w_r = \frac{e_r}{\sqrt{1 + x_r'(X_{r-1}'X_{r-1})^{-1}x_r}}$$

The CUSUM of squares test uses

$$S_t = \frac{\sum_{r=K+1}^{t} w_r^2}{\sum_{r=K+1}^{T} w_r^2} \tag{4.12}$$

Since the residuals are independent, each of the two terms is approximately a sum of chi-square variables each with one degree of freedom. Therefore, $E[S_t]$ is approximately $(t-K)/(T-K)$. The test is carried out by constructing confidence bounds for $E[S_t]$ at the values of t and plotting S_t and these bounds against t. The appropriate bounds are $E[S] \pm c_0$, where c_0 depends on both $(T-K)$ and the significance level desired. As before if the cumulated sum strays out the confidence bounds, doubt is cast on the hypothesis of parameters stability. This test was applied in the fits provided by (4). Results are shown in table 1 where 0 represents the validity of the null hypothesis (constancy of parameter) and 1 indicates that coefficients do not remain constant during the full sample period at 95% of significance. The null hypothesis of stability was rejected in 29 cases.

4.6.3 Moving Regressions

The relationship between long-run elasticities and the traffic level shows that high traffic level motorways tend to have smaller elasticities and the cusum of squares test show that parameters may be varying over time. The link between these two results will be to show that within each section, the elasticity is decreasing. A simple diagnostic test to detect the decreasing of the parameter is to partition the sample into subsamples of approximated equal number of observations each. We set 2 subsamples of approximately 15 years (with overlapping). Results in table 4.5 (ss_1 and ss_2 for subsamples 1 and 2 respectively) show that a globally decreasing elasticity can be observed in all but 2 sections, and in most cases, the elasticity in the second period is also smaller than the lower bound (95%) of the first subsample.

4.7 A Functional Form for Decreasing Elasticity

There are different ways to specify declining elasticities. Some studies (as in Dargay et al. (2002) propose “inconditional” declining elasticites by replacing the log of GDP by the inverse of some function of GDP (GDP, $ln(GDP)$, or other). Dargay et al. (2002) find that declining elasticities are more arguable and provide statistically better fits.

Precedent results and the theoretical arguments explained before lead us to consider a variable relation between traffic and economic growths by an elasticity depending on the traffic level. To take in account the asymptotically decreasing put in evidence, we propose the following formulation:

$$\varepsilon_{T/GDP}(T) = \frac{\frac{\delta T}{T}}{\frac{\delta GDP}{GDP}} = kT^{\gamma} \tag{4.13}$$

where k is a positive constant and γ is a negative constant. The parameter γ may be interpreted as the elasticity of the - elasticity of traffic with respect to the GDP - with respect to the traffic level, since:

$$\varepsilon_{\varepsilon_{T/GDP}/T} = \frac{\delta\varepsilon_{T/GDP}}{\delta T}\frac{T}{\varepsilon_{T/GDP}} = \gamma k T^{\gamma-1}\frac{T}{kT^{\gamma}} = \gamma$$

The differential equation (4.13) is separable and its solution (for $\gamma \neq 0$) is:

$$T = (-\gamma(klnGDP + c))^{-\frac{1}{\gamma}} \tag{4.14}$$

Where c is the constant from the integration. Assuming that this relation holds for the first period (T_1, GDP_1) and both T_1 and GDP_1 are normalized to one then T becomes:

$$T = (1 - \gamma klnGDP)^{-\frac{1}{\gamma}} \tag{4.15}$$

The equation (4.3) can be therefore rewritten as:

$$lnT_{it} = \beta_{0i} - \frac{1}{\gamma_i}ln(1 - \gamma_i k_i lnGDP_t) + \alpha_{2i}lnPF_t + \alpha_{3i}lnToll^{M}_{it} + \varepsilon_{it} \tag{4.16}$$

This approach sets up an intrinsic relation between the traffic level and its reactivity to economic growth, as wanted; it allows for a good representation of the phenomenon and an easy interpretation of results at the cost of introducing a non-linearity in the transport demand equation.

Estimated γ and k are reported in appendix 1. Results provide very good fits and proper values, except in two cases, for which we estimated positives values for γ (for the same sections where the moving regressions indicated a growth instead of a decreasing of the elasticities), indicating that the maturity has not been reached; these values shall be used with care for forecast purposes. Figure 5 compares the constant and the variable elasticity for section 40; the vertical line represents the ratio between the traffic in the last and in the first periods.

Figure 4.9: Comparing elasticities.

We could expect lower traffic motorways to have higher k's and higher γ's, this result is confirmed in our analysis; it can be graphically viewed in figure 6 (we do not include the two positive values of γ and their respective k). We can observe also that the dispersion of γ increases with the traffic level. This result means that high traffic motorways may be at different stages of maturity, as we could expect.

The same principle can be applied to the PAM and to the ECM. For these models we can apply two different approaches. The first one consists in setting a decreasing parameter for the GDP, as for the LTM. This will nevertheless imply a decreasing short-run elasticity for the PAM. The second approach is,

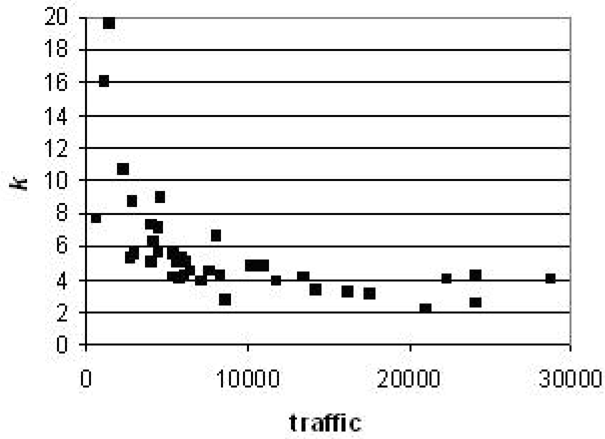

Figure 4.10: k versus traffic.

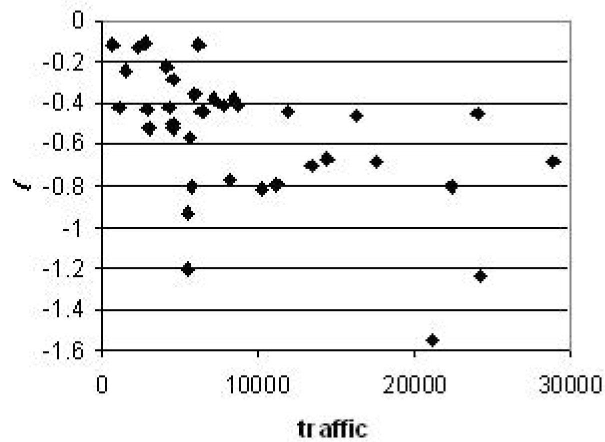

Figure 4.11: γ versus traffic.

instead of setting a decreasing coefficient with respect to the GDP, consider a growth of the adjustment coefficient (θ in the PAM and -1 in the ECM) following the same pattern. This formulation leads to the same results in terms of long-run elasticities and is consistent with the economic intuition behind the hypothesis of decreasing elasticity.

Writing $\phi = kT^{\gamma}$ in the PAM, equation (4.7) becomes:

$$lnT_{it} = \beta_{0i} + \alpha_{1i} lnGDP_t + \alpha_{2i} lnPF_t + \alpha_{3i} lnToll^M_{it} - \frac{1}{\gamma_i} ln(1 - \gamma_i k_i lnT_{t-1}) + \varepsilon_{it} \tag{4.17}$$

and the long-run elasticities will be given by the ration of the short-run value by $1 - k_i T^{\gamma_i}$, where $0 < k_i < 1$ and $\gamma_i < 0$.

Making $\varphi - 1 = kT^{\gamma}$ (where k will be negative and γ positive) the ECM (4.11) can be re-written as:

$$\begin{aligned} \Delta lnT_{it} &= \beta_{0i} + \beta_{1i} \Delta lnGDP_t + \beta_{2i} \Delta lnPF_t + \beta 3i \Delta lnToll^M_{it} \\ &- \frac{1}{\gamma_i} ln(1 - \gamma_i k_i lnT_{t-1}) + \alpha_{2i} lnGDP_{t-1} + \alpha_{3i} lnPF_{t-1} + \alpha_{4i} lnToll^M_{it-1} + \varepsilon_{it} \end{aligned} \tag{4.18}$$

The long-run elasticities will be given by $\alpha / -k_i T^{\gamma_i}$ or equivalently, $-\alpha k_i T^{-\gamma_i}$.

4.7.1 Impact on Long-Term Forecasts

As we can see in figure 5, if the elasticity decreases with the traffic growth, the assumption of a constant elasticity will tend to overestimate the future traffic. Consider the hypothetical case in figure 7a where both initial traffic is GDP are normalized to 1, the constant elasticity is 2.0, $k = 2.5$ and $\gamma = -0.5$. We can see that in the short term results from both models are very close. As the GDP increases the difference becomes more important; the classic model presents a globally convex profile while the new model produces a concave evolution.

This approach was applied in a large scale forecast traffic until 2030 to the main French private motorways. One example is given in the figure 7b; both models presented very good fits ($R^2 > 0.98$). Results show that the variable elasticity model produces more conservative forecasts. Moreover, estimating both models using data until 1999 and comparing the forecasts between 2000 and 2005 with the actual traffic we can see that the variable elasticity model was twice more precise.

This method is however very data greedy. If no information on parameters

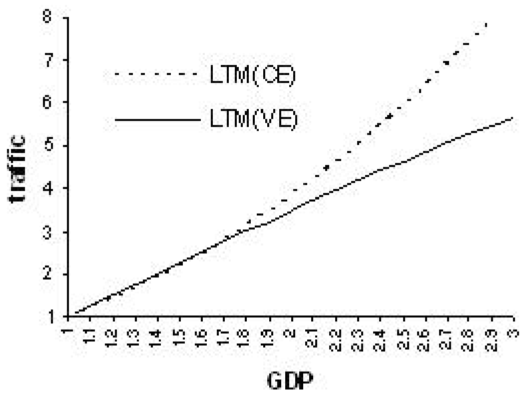

Figure 4.12: A hypothetical example.

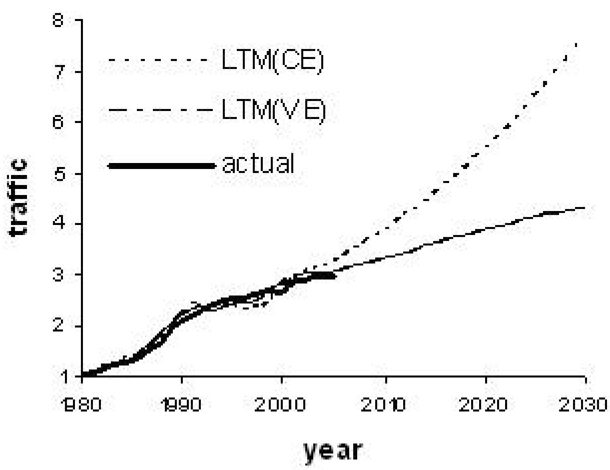

Figure 4.13: Application on the A11 motorway.

is inferred, a quite long data series is needed to calibrate the model but it confers a significant advantage in terms of results for very long term forecasts for which the constant elasticity seems to be an unrealistic and overoptimistic hypothesis.

4.8 Conclusions

In this chapter we put in evidence the decreasing of the elasticity of traffic with respect to the GDP, which characterises the traffic maturity and have shown that the hypothesis of constant elasticity assumed by classic models is unrealistic and leads to traffic overestimation. A new model of decreasing elasticity is proposed setting up an intrinsic relation between the traffic level and its reactivity to economic growth. This model allows for a good representation of the phenomenon, a good interpretation of results and gives a rigorous econometric approach to time-series traffic forecasts, at the cost of introducing a non-linearity in the equation. In the short term the model results are closer to that given by the classical constant elasticity model; in the long term, where classic models tend to produce linear or convex profiles, this model reproduces the observed concavity. This model allows for a better interpretation of the coupling between traffic and economic growth, and a more accurate long-term forecast.

Table 4.2: ADF test - traffic

	Variables (in logarithms)		LTM residuals	
	adf	p-value	adf	p-value
section 1	-2.4394	0.405	-1.3287	0.8281
section 2	-1.3288	0.828	-1.5716	0.7356
section 3	-2.9603	0.2065	-2.228	0.4855
section 4	-1.1303	0.9014	-1.467	0.7754
section 5	-1.7939	0.6509	-1.1409	0.8997
section 6	-0.6814	0.9599	-1.371	0.812
section 7	-5.9499	0.01	-1.7975	0.6495
section 8	-2.2077	0.4933	-3.1229	0.1446
section 9	-1.7048	0.6848	-3.3294	0.08758
section 10	-2.9225	0.2209	-2.0562	0.551
section 11	-0.6509	0.9624	-2.3594	0.4355
section 12	-1.9304	0.5989	-1.4336	0.7882
section 13	-2.9601	0.2066	-2.4367	0.406
section 14	-2.2191	0.4889	-2.3243	0.4489
section 15	-1.8089	0.6452	-2.8079	0.2646
section 16	-2.4413	0.4043	-2.4306	0.4083
section 17	-1.3369	0.825	-1.464	0.7766
section 18	-1.6953	0.6885	-1.9466	0.5927
section 19	-1.7911	0.652	-3.184	0.1213
section 20	-2.4587	0.3977	-2.6592	0.3213
section 21	-2.3947	0.422	-1.8303	0.637
section 22	-1.552	0.743	-2.4978	0.3828
section 23	-2.756	0.2844	-1.8363	0.6348
section 24	-2.1455	0.5169	-2.2191	0.4889
section 25	-3.2599	0.09723	-2.5728	0.3542
section 26	-2.235	0.4828	-1.5475	0.7448
section 27	-2.4379	0.4056	-1.9891	0.5765
section 28	-1.1658	0.8902	-1.8397	0.6334
section 29	-3.2201	0.1076	-1.1729	0.8875
section 30	-2.5795	0.3516	-1.8774	0.6191
section 31	-2.156	0.513	-1.7083	0.6835
section 32	-2.5759	0.353	-1.6994	0.6869
section 33	-1.4993	0.7631	-1.4238	0.7919
section 34	-2.133	0.5217	-3.3224	0.08856
section 35	-1.3087	0.8357	-1.7997	0.6487
section 36	-1.0752	0.9095	-1.6362	0.711
section 37	-1.5235	0.7539	-2.1963	0.4976
section 38	-0.9742	0.9244	-1.4389	0.7861
section 39	-1.1471	0.8973	-1.6602	0.7018
section 40	-0.6956	0.9587	-2.2377	0.4818

Table 4.3: Summary of descriptive statistics

ID	L	year0	traffic0	elt(LTM)	esr(PAM)	elr(PAM)	esr(ECM)	elr(ECM)
1	25	1980	21090	1.15	0.65	1.24	0.71	1.38
2	18	1987	2362	6.03	0.82	2.69	(1.03)	(2.86)
3	25	1980	24164	1.84	0.42	2.50	(-0.07)	(0.42)
4	25	1980	6177	4.17	1.95	5.84	(1.41)	4.60
5	25	1980	5499	1.95	0.54	2.44	(0.16)	(1.37)
6	22	1983	4630	5.02	1.76	5.34	1.69	(3.39)
7	22	1983	662	6.71	(1.26)	(9.56)	(0.66)	(7.86)
8	20	1985	1532	9.35	(0.39)	(1.27)	(1.02)	6.48
9	25	1980	13456	2.37	0.62	2.32	0.58	(1.47)
10	25	1980	7541	2.43	0.45	2.01	1.29	1.94
11	25	1980	6002	3.54	0.83	3.88	(0.19)	2.23
12	25	1980	6296	3.23	1.37	3.48	0.95	3.20
13	25	1980	4505	4.11	1.90	5.17	(0.95)	4.40
14	25	1980	24111	2.00	1.15	2.18	(0.68)	2.15
15	25	1980	4332	3.76	1.09	4.47	1.18	3.78
16	25	1980	16252	2.35	0.96	2.52	(0.56)	2.34
17	25	1980	8709	2.04	(0.38)	(1.95)	(0.63)	1.89
18	25	1980	2917	4.43	(0.26)	(2.09)	1.44	2.32
19	25	1980	2768	4.51	1.13	3.69	(0.81)	3.33
20	25	1980	6565	2.94	0.86	2.93	(0.75)	2.37
21	24	1981	8370	3.11	1.21	3.23	1.05	2.60
22	18	1987	6494	2.97	0.86	2.22	(-0.90)	2.22
23	25	1980	28854	2.34	0.80	2.67	1.01	(2.55)
24	25	1980	11130	2.19	0.79	2.81	0.63	(2.47)
25	25	1980	4146	3.70	1.07	3.85	2.21	(4.27)
26	25	1980	10236	2.33	0.73	3.02	0.98	2.95
27	25	1980	4159	4.92	1.75	5.03	3.04	4.11
28	25	1980	5507	2.40	0.26	2.62	(0.32)	2.25
29	25	1980	17540	2.42	1.59	2.47	1.39	2.42
30	25	1980	14332	2.28	1.16	2.51	0.75	2.41
31	19	1986	5835	2.14	0.32	1.37	(-0.54)	1.41
32	25	1980	22402	2.19	0.72	2.63	(0.55)	2.00
33	25	1980	7162	2.73	0.88	3.33	(0.42)	3.07
34	25	1980	3074	3.18	(0.46)	(3.88)	(-0.19)	(2.35)
35	23	1982	1138	6.94	1.45	5.89	(1.31)	6.83
36	25	1980	8130	2.67	(0.34)	(3.21)	0.73	(-0.18)
37	25	1980	4496	3.37	0.62	4.49	(0.59)	(0.44)
38	25	1980	7777	2.73	0.90	3.70	(1.00)	3.38
39	25	1980	5700	2.71	0.74	4.15	1.07	4.15
40	25	1980	11834	3.04	1.17	3.33	0.87	3.04

Table 4.4: CUSUM of squares test

section	$cusum^2$	section	$cusum^2$
1	1	21	1
2	1	22	0
3	1	23	1
4	0	24	1
5	1	25	0
6	1	26	1
7	1	27	1
8	0	28	1
9	1	29	1
10	0	30	1
11	0	31	1
12	1	32	1
13	0	33	0
14	1	34	1
15	1	35	1
16	0	36	1
17	0	37	1
18	1	38	1
19	0	39	1
20	1	40	1

Table 4.5: Subsamples Elasticities

section	e_{ss1}	e_{ss2}	$e_{ss2} < e_{ss1}$	section	e_{ss1}	e_{ss2}	$ess2 < ess1$
1	1.39	0.42	1	21	3.36	1.98	1
2	9.36	2.09	1	22	2.05	2.26	0
3	2.26	0.59	1	23	2.89	1.03	1
4	4.29	1.77	1	24	3.05	0.91	1
5	2.43	1.42	1	25	3.46	2.02	1
6	5.08	3.62	1	26	3.13	0.88	1
7	9.26	3.98	1	27	5.34	1.53	1
8	11.31	2.19	1	28	3.41	0.87	1
9	2.44	1.51	1	29	2.60	2.35	1
10	2.26	2.54	0	30	2.52	2.17	1
11	4.08	1.68	1	31	2.64	1.40	1
12	3.94	2.16	1	32	2.49	1.26	1
13	5.07	1.87	1	33	2.98	1.34	1
14	2.21	1.65	1	34	3.64	1.55	1
15	4.44	2.44	1	35	7.17	2.11	1
16	2.58	2.01	1	36	3.36	1.24	1
17	2.18	2.15	1	37	4.12	1.55	1
18	5.33	2.22	1	38	3.16	1.52	1
19	4.81	2.73	1	39	3.15	1.33	1
20	3.29	2.26	1	40	2.84	1.55	1